45-60 min

13+ years

Superposition with Cards

Explore quantum superposition of qubits

Learning Goals

Students will:

- Understand that qubits can be in a superposition state of both 0 and 1 at the same time.
- Understand that qubits in a superposition state have a certain probability of being measured as 0 or 1.
- Understand that once measured, a qubit will be in the state corresponding to the measured outcome.
- Interpret and construct simple models of superposition states

Importance in Quantum Computing

Quantum computers store and process information in the form of quantum bits (or qubits). A qubit can exist in a **superposition** of 0 and 1 at the same time. Once measured, the superposition state collapses, and the qubit's state will be either 0 or 1.

Materials

- ☐ ***Superposition with Cards*** slide deck
- ☐ ***Superposition with Cards*** worksheet
- ☐ Sets of 20 playing cards (10 red/10 black) for each group of 4 students

Preparation

- Arrange desks so that students can easily work in groups of 4
- Download and check that slides can be projected properly

Background Knowledge

A quantum computer is a new type of computer that stores and processes information in the form of quantum bits (or qubits) rather than bits like a classical computer (i.e., an ordinary, non-quantum computer). An important feature of qubits is that they can exist in a **superposition** of 0 and 1 at the same time. This is in contrast to classical bits which can only have one of two possible values, 0 or 1. When a qubit is in a superposition state, it has a certain probability of being measured as 0 and a certain probability of being measured as 1. When a qubit is measured, we observe the result (either 0 or 1), and the state immediately "collapses" into the state corresponding to the outcome that was observed. Quantum computers are able to use superposition to store and process information in a way that is difficult or impossible for classical computers.

Facilitating the Activity

ENGAGE

Use the slides in the *Superposition with Cards* slide deck to introduce the concepts in this opening discussion.

SLIDE 2: What are classical bits?
Tell students that modern computers use **classical bits** to store information. Classical bits store information by storing either the value 0 or 1.

SLIDE 3: ASK: How many possible states can a classical bit have? *(Answer: 2)*

SLIDE 4: Introduce students to modeling classical bits with cards:

- Red cards will represent 0
- Black cards will represent 1

SLIDE 5: ASK: "What is the state of this *classical bit?" (Answer: 0)*

SLIDE 6: Share that **probability** indicates how likely something is to happen.
ASK: What is the probability of getting **HEADS**? *(Answer: ½)*
ASK: What is the probability of getting **TAILS**? *(Answer: ½)*

SLIDE 7: ASK: What is the probability of rolling a **2**? *(Answer: ⅙)*
ASK: What is the probability of rolling an **EVEN** number? *(Answer: ½)*
ASK: What is the probability of rolling a **PRIME** number? *(Answer: ½)*

SLIDE 8: Discuss basic properties of probability.

- The sum of all probabilities should always be 1
- A probability of 1 indicates something will ALWAYS happen
- A probability of 0 indicates something will NEVER happen
- If A has a greater probability than B, then A is more likely to happen

SLIDE 9: What are Qubits?

- Quantum computers are a new kind of computer that works completely differently than classical (non-quantum) computers
- Quantum computers use quantum bits (qubits) to store information
- Qubits can be in a superposition state, holding both 0 & 1 at the same time!
- Measuring a qubit, collapses the qubit's state to either 0 or 1

SLIDE 10: Qubits in Superposition

- Share with students that qubits can be in a state of **superposition.** They do not always have to be either 0 or 1, but could be both at the same time.
- When in superposition, qubits have a probability of being 0 and a probability of being 1. (These two probabilities must sum to 1.)
- **ASK**: Which of these qubits has the greatest chance of being 0? *(Answer: B)*
- **ASK**: Which of these qubits has the smallest chance of being 0? *(Answer: C)*
- Emphasize that classical bits CANNOT be in superposition. They can only be 0 or 1.

SLIDE 11: Measurement

- The purpose of measuring a qubit is to find out if it has the value 0 or 1.
- After measuring a qubit, it collapses the state into either 0 or 1, depending on what value was observed.
- Describe the example where the value 0 is observed.
- **ASK**: If you measure the qubit and observe the value 1, what do we get if we measure the qubit again? (*Answer: The qubit collapses into the 1 state, so we will get 1 again with 100% probability.)*

ACTIVITY

Modeling a Superposition State with Cards

Have students form groups of 4. Distribute the **Superposition Worksheet** to students and sets of cards to groups.

Review the model with students:

ASK: "Hold up a card that represents the state 0." *(This card should be RED.)*
ASK: "Hold up a card that represents the state 1." *(This card should be BLACK).*

Introduce modeling a superposition state with cards (example is shown on worksheet):

SAY: "A qubit in **superposition** can be represented by a pile of RED and BLACK cards, placed face down. **Probabilities** are determined by the ratio of RED cards to BLACK cards."
Hold up 1 RED card and 1 BLACK card, and shuffle the cards while face down.
SAY: This represents a qubit in a state of superposition and that the qubit has a 50% chance of being either 0 or 1

ASK: "Why does this set of cards represent a 50% chance of getting either 0 or 1?" *(The ratio of RED:BLACK cards is 1:1)*

Have students form a qubit that has a 66% chance of 0 and 33% chance of 1.
Then, ask the following questions:

ASK: "How many cards are in your pile?"
ASK: "How many cards of each color are in your pile?"
ASK: "Why did you choose those cards?"

Students should begin filling in their worksheet as you work through the example as a whole group. Students will note the combinations of RED/BLACK cards used in each pile (e.g., 66% : 0; 33% : 1 → RRB), as well as the number of each color of card.

Measurement

Hold up your pile for a 50%/50% qubit state with the cards face down.

SAY: "To **measure** a qubit in a superposition state, shuffle the pile and flip over the top card. That card represents the new state of the qubit."
ASK: "What is the probability of measuring a 1?" (Answer = 50%)

Shuffle the cards face down, flip over the top card, and show the card to students.

ASK: "This is the new state of the qubit. Is it still a superposition?" *(Answer: No)*

ASK: "What is the new state of the card? *(Answers will vary based on selected card)*

ASK: "Now, if the qubit is measured again, what is the probability of measuring red/black?" *(If qubit was measured as RED: 0 with probability 100%. If qubit was measured BLACK: 1 with probability 100%)*

SAY: The superposition "**collapsed**" into one state or the other. *(If you got a RED card, then it "collapsed" into the 0 state. If you got a BLACK card, then it "collapsed" into the 1 state.)*

Working in groups, students complete the worksheet: (1) modeling the remaining qubit states on the front of the worksheet, and then (2) measuring a single qubit state 12 times, noting the outcome each time. Emphasize that each time the qubit is measured, the qubit needs to be reset back to a superposition state.

DISCUSSION

Have students share out their results.

ASK: "Were the results surprising, or did they confirm what you were expecting?"

ASK: "Given the probabilities associated with the qubit, how many 0s and 1s would you expect?" *(Answer: THREE 1's & NINE 0's)*

ASK: "How many groups got approximately <u>three</u> 1's and <u>nine</u> 0's?"

ASK: "Does it make sense that some groups got <u>more than three</u> 1's or <u>fewer than three</u> 1's for their measurements results?" *(Yes. Each measurement is independent from the others and has the possibility of resulting in 0 or 1. While unlikely, it is even possible for a qubit to have the same measurement outcome for all 12 trials!)*

(FOR HIGH SCHOOL STUDENTS) Discuss the problem of measuring a stack of cards containing an unknown number of 0's and 1's. If you have the time, consider having students try it out a few times.

ASK: "Can you estimate the ratio of 0's and 1's after one measurement?"

ASK: "Can you estimate the ratio after multiple measurements?"

ASK: "Is your estimate improving with each measurement?"

ASK: "How would you do this experiment?"

Connections to Standards

Next Generation Science Standards*

Crosscutting Concept: Patterns, Cause and Effect

NGSS Science and Engineering Practices: Using Mathematics and Computational Thinking, Constructing Explanations, Engaging in Argument from Evidence

Common Core State Standards

S-IC.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S-IC.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

QIS K-12 Key Concepts

3. Quantum applications are designed to carefully manipulate fragile quantum systems without observation to increase the probability that the final **measurement** will provide the intended result.

(3.a) A measurement is an interaction with the quantum system that transforms a state with multiple possible outcomes into a "collapsed" state that now has only one outcome: the measured outcome.

(3.b) A quantum state determines the probability of the outcome of a single quantum measurement, but one outcome rarely reveals complete information about the system.

4. The **quantum bit, or qubit**, is the fundamental unit of quantum information, and is encoded in a physical system, such as polarization states of light, energy states of an atom, or spin states of an electron.
(4.a) Unlike a classical bit, each qubit can represent information in a **superposition**, or vector sum that incorporates two mutually exclusive quantum states.

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